THE MOVING FORCE PROBLEM REVISITED

N. Wagner, R. Helfrich
(INTES GmbH, Germany)

Abstract

Moving loads are quite common in real life applications and are wide-spread over various engineering disciplines with an increasing level of detail nowadays. Civil engineers are mainly interested in the vehicle-bridge interaction problem, which is the most extensively studied type of moving load problem. Besides that train-track interaction, vehicle-road/ground interactions are investigated. Ouyang [5] listed various moving load problems such as flexible discs, rotating beam/shafts and spindles, cranes, strings, and shells subjected to moving loads in a recent review article. The main difficulty to deal with moving mass problems in commercial finite element solvers is the fact that the system matrices of the second-order differential equation are inherently time-varying. Although one should be aware of the fact that removing the time-varying character of the system matrices introduces some kind of error the assumption of a moving force is valid especially for low speeds. However, it is well known that this error increases with velocity. In addition, stability issues due to time-periodic crossings by a series of moving masses cannot be handled by the moving force approach. Therefore the moving mass problem is usually simplified by a moving force problem in order to reduce the numerical complexity of the problem. Thus only the right hand side of the equations of motion is affected by the travelling load. An example of a circular arch traversed by a moving force is used to demonstrate the procedure using the commercial finite element package PERMAS. The trajectory of the moving force is prescribed and the coupling itself is realized by a multipoint constraint. The displacement results of a beam model taken from the literature and a solid model used in the current study are compared. It appears that both approaches are in good agreement and justify the moving force approach in a first stage.

1. Overview

Dynamic characteristics of structures due to various moving forces is an important problem in engineering. A good overview of applications related to moving load problems is given by Ouyang [5]. Wu [12] investigated the vibration of a rectangular plate undergoing forces moving along a circular path. Experimental investigations of a multi-span flexible structure subjected to moving masses are conducted by Stancioiu [9]. Laminated composite plates traversed by a moving mass are studied by Ghafoori [2] and the moving oscillator by Mohebpour [4]. From a numerical point of view, the moving mass problem is more challenging than the moving force problem. This becomes even clearer if one takes a closer look at the underlying equations of motion. Special time-stepping procedures are developed for time-varying differential equations [11,14]. The moving mass problem is discussed in [1].The moving oscillator problem is tackled in [3,4,8] and for functionally graded simply supported Euler-Bernoulli beams in [6]. The separation and reattachment of moving oscillators is considered by Ouyang et. al. [8]. All FEM computations are carried out in PERMAS. PERMAS specific commands are highlighted by a preceding dollar sign and capital letters in the subsequent sections.

2. Equations of motion

The equations of motion for the moving force problem take the form

$$M\,\ddot{u} + D\,\dot{u} + K\,u = f(t)\ , \tag{1}$$

where M, D, K are, respectively, the overall mass, damping and stiffness matrices, $u(t)$ is the displacement vector and $f(t)$ is the external force vector. The system matrices are time-invariant in that case.

The moving mass problem is characterized by a linear time-varying system

$$M(t)\ddot{u} + D(t)\dot{u} + K(t)u = f(t)\ \delta(x - vt);\quad f(t) = mg - m\ \frac{\mathrm{d}^2 w(x,t)}{\mathrm{d}t^2} \tag{2}$$

due to the fact, that the inertia forces

$$m\ \frac{\mathrm{d}^2 w(x,t)}{\mathrm{d}t^2} = m\left(\frac{\partial^2 w}{\partial t^2} + 2\,v\ \frac{\partial^2 w}{\partial x\,\partial t} + \ v^2\ \frac{\partial^2 w}{\partial x^2} + \dot{v}\ \frac{\partial w}{\partial x}\right) \tag{3}$$

are taken into account [11]. The influence of the different terms, i.e. conventional vertical inertia force, Coriolis force and centripetal force in (3) on the dynamic response is studied by Sharbati [7]. The last term in (3) vanishes for constant speeds.

The moving load problem needs a special treatment in PERMAS. The DIRECT NLTIME procedure in combination with a $MPC UPDATE definition within the constraint variant is needed to capture the effect of the time-varying position of the load. The coupling of the moving load with the supporting structure is realized by a $MPC ISURFACE definition in the model file. Furthermore, we assume that the moving load is in permanent contact with the supporting structure. Thus a possible separation is not considered here and can be found in Stancioiu [8].

Finally the moving oscillator problem is considered. In that case, the moving mass is attached to the structure either by a spring or by a mass-spring assembly. When the oscillator slides over a straight beam, the coupled equations of motion are described by [8]

$$EI \frac{\partial^4 w(x,t)}{\partial x^4} + \varrho A \frac{\partial^2 w(x,t)}{\partial t^2} = -f_c(x,t)\delta(x - vt) \quad , \tag{4}$$

$$m_s \ddot{z} = -k(z-u) - c(\dot{z} - \dot{u}) - m_s g, \tag{5}$$

$$m_u \ddot{u} = k(z-u) + c(\dot{z} - \dot{u}) - m_u g + f_c(vt,t) \tag{6}$$

$$u(t) = w(x,t)\,|_{x=vt} \quad , \qquad f_c(vt,t) > 0\,. \tag{7}$$

It can be shown, that the moving oscillator problem tends towards the moving mass problem for an infinite stiffness of the oscillator.

3. Numerical Examples

The first example is taken from Wu [12]. The finite element model is depicted in Fig. 1 and consists of 128 shell elements. Boundary conditions of the hinged-hinged plate are depicted by red, green and blue arrows corresponding to the direction of the constraints. The circular path is visualized by so-called plot elements.

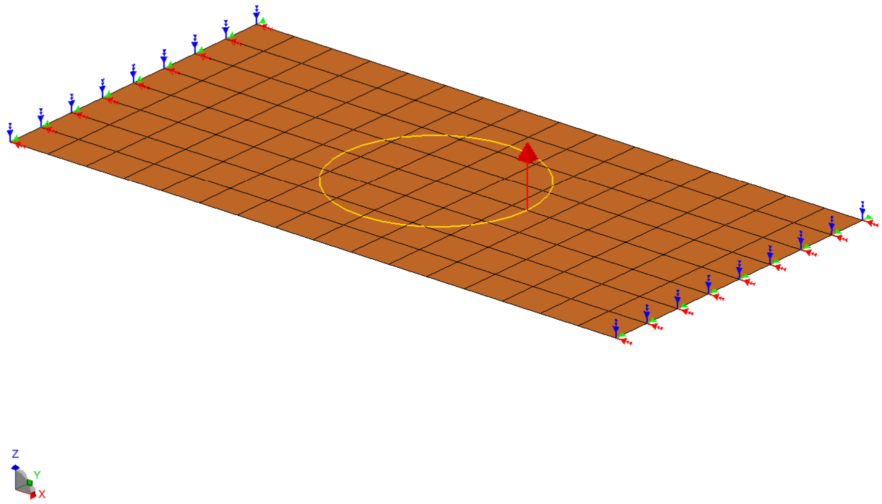

Figure 1: Rectangular plate subjected to a harmonic force (denoted by a red arrow) moving along a circular path.

Table 1: Geometrical and physical properties of the rectangular plate

Length L_x [m]	2.0
Width L_y [m]	1.0
Thickness t [m]	0.01
Young's modulus E [GPa]	206.8
Density ϱ [kg/m^3]	7820.
Poisson's ratio ν	0.29
Radius R [m]	0.3

A concentrated harmonic force

$$f_z(t) = \begin{cases} \text{f} \sin \Omega\, \text{t} & ; \text{t} \leq 10 \text{ [s]} \\ \text{f} \sin 10\, \Omega ; & \text{t} > 10 \text{ [s]} \end{cases} \tag{8}$$

with an excitation frequency Ω moves along a circular path with radius r and center $C = [x_c, y_c]$ with constant rotating speed ω. The center C is coincident with the center of gravity of the rectangular plate. The current position of the moving force is realized by a prescribed motion of the corresponding node

$$r_p(t) = r(t) - r_0 ; \qquad r(t) = r_C + R \begin{bmatrix} \cos \omega\, t \\ \sin \omega\, t \end{bmatrix}, \qquad r_0 = r(t = 0), \tag{9}$$

where ω denotes the rotational speed of the moving force. The influence of the excitation frequencies ω, Ω on the response is illustrated in Fig. 2 and Fig. 3 for two different values. The first natural frequency of the hinged plated is $\omega_1 = 2\pi f_1 = 37.1066\ [\frac{\text{rad}}{\text{s}}]$; the second is given by $\omega_2 = 107.201\ [\frac{\text{rad}}{\text{s}}]$. The time for one orbit is $T = \frac{2\,\pi}{\omega}$. The response of the center node is presented in Fig. 2 for a resonance excitation. In the second case, an excitation frequency between the first and second eigenfrequency is used. Other studies show similar results [12, 13].

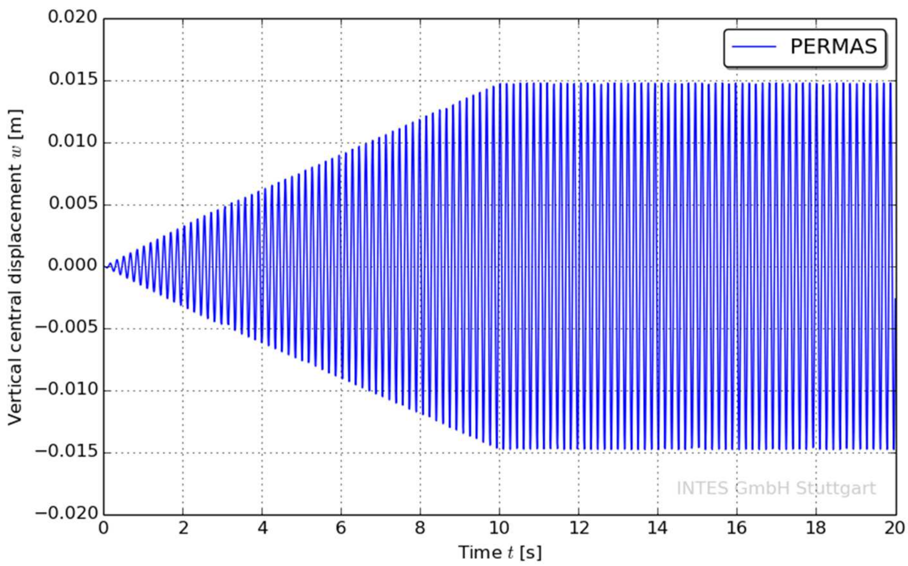

Figure 2: Time history for the vertical displacements of the centre of the hinged plate subjected to a single concentrated sinusoidal force moving along a circular path with a constant rotating speed $\omega = \Omega = 37.061\ [\frac{\text{rad}}{\text{s}}]$.

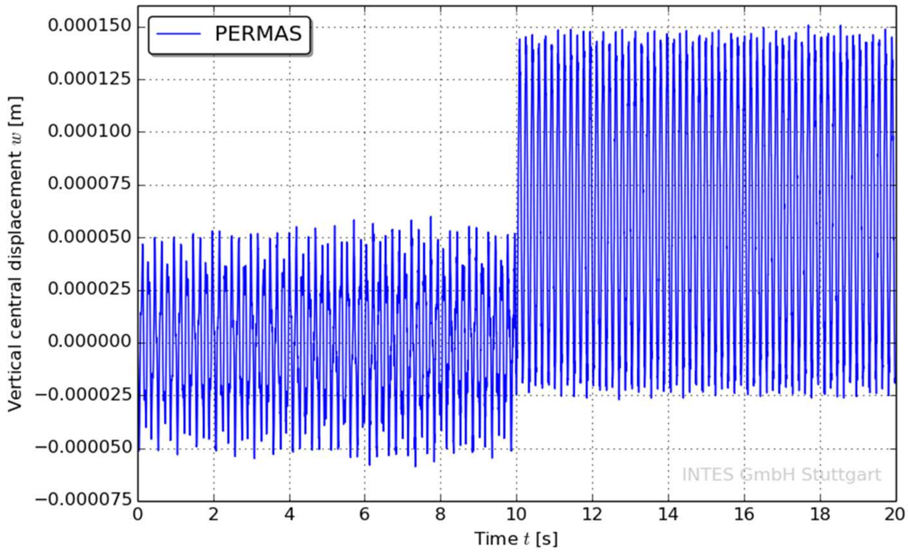

Figure 3: Time history for the vertical displacements of the centre of the hinged plate subjected to a single concentrated sinusoidal force moving along a circular path with a constant rotating speed $\omega = \Omega = 99.195\ [\frac{\mathrm{rad}}{\mathrm{s}}]$.

Table 2: Geometrical and physical properties of the horizontally curved beam

Cross-sectional area $A = ab$ [m^2]	5*1.8
Total arc length $L = R\,\bar{\alpha}$ [m]	45.84*$\pi/6$
Young's modulus E [GPa]	32.2
Poisson's ratio ν	0.2
Velocity v_p [m/s]	40.0
Force P [N]	293020.

The second example is taken from Yang [14] where curved beam elements have been used. In this study, the finite element model illustrated in Fig. 4 consists of 800 hexahedral elements. The properties are listed in Table 2.

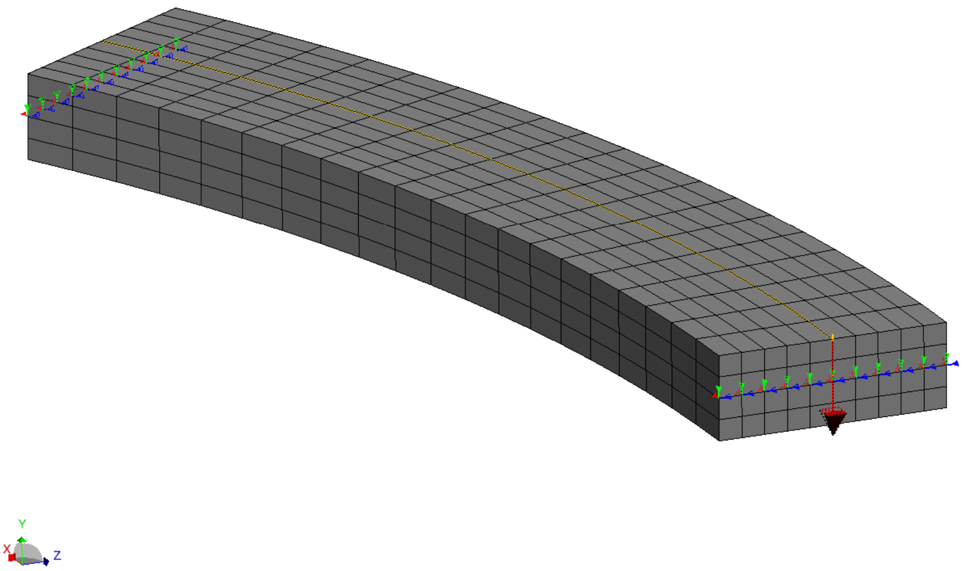

Figure 4: Horizontally curved beam subjected to a moving load P.

The dynamic response of the horizontally curved beam subjected to a constant moving force with $P = 293020\,[\mathrm{N}]$ is depicted in Fig. 5. A perfect agreement of previously published results [13,14] is achieved. In addition, Fig. 6 depicts the vertical displacements of the upper beam surface on the middle line over time for all nodes on this line simultaneously.

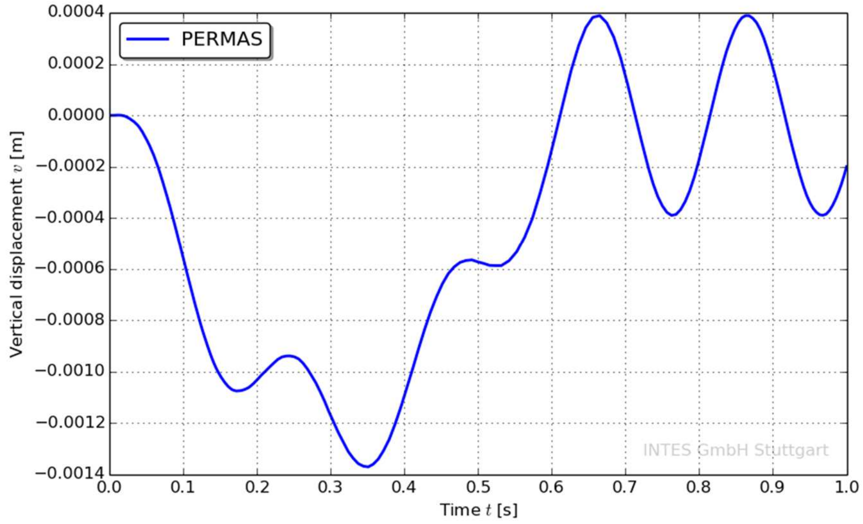

Figure 5: Time history of vertical displacement at the middle point C of the simply supported horizontally curved beam subjected to a moving load.

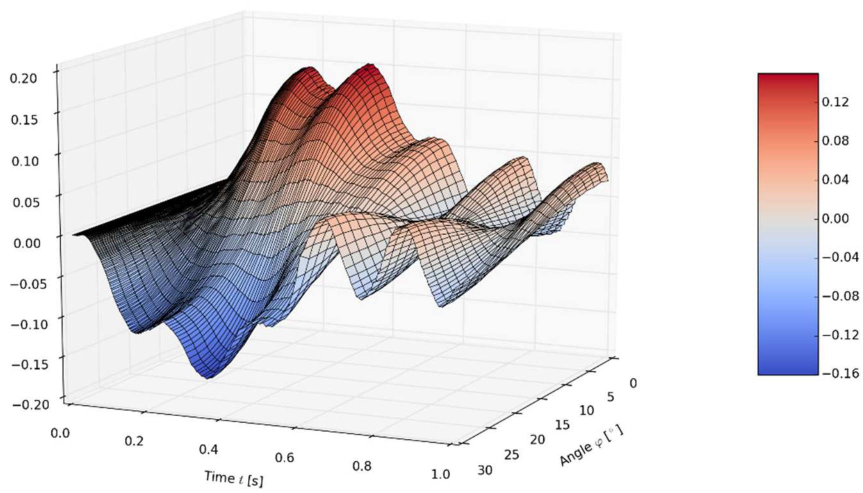

Figure 6: Vertical displacement field $v(\varphi, t)$ [mm] of the simply supported horizontally curved beam subjected to a moving load along the angular coordinate.

4. Conclusions

Several examples from the literature are used to validate the procedure that is implemented in PERMAS for the moving force problem. Complex trajectories of the moving force can be easily implemented by a comprehensive function library.

5. References

[1] Esen, Ismail (2013), A new finite element for transverse vibration of rectangular thin plates under a moving mass, Finite Elements in Analysis and Design, Vol. 66, pp. 26—35.

[2] Ghafoori, E.; Asghari, M. (2010), Dynamic analysis of laminated composite plates traversed by a moving mass based on a first-order theory, Composite Structures, Vol. 92, pp. 1865—1876.

[3] Hassanabadi, M. E.; Amiri, J. V.; Davoodi, M. R. (2014), On the vibration of a thin rectangular plate carrying a moving oscillator, Scientia Iranica A, Vol. 21, pp. 284—294.

[4] Mohebpour, S.R.; Malekzadeh, P.; Ahmadzadeh, A.A. (2011), Dynamic analysis of laminated composite plates subjected to a moving oscillator by FEM, Composite Structures, Vol. 93, pp. 1574—1583.

[5] Ouyang, H. (2011), Moving-load dynamic problems: a tutorial (with a brief overview), Mechanical Systems and Signal Processing, Vol. 25, pp. 2039—2060.

[6] Rajabi, K.; Karganovin, M. H.; Gharini, M. (2012), Dynamic analysis of a functionally graded simply supported Euler-Bernoulli beam subjected to a moving oscillator, Acta Mechanica,

[7] Sharbati, E.; Szyszkowski, W. (2011), A new FEM approach for analysis of beams with relative movements of masses. Finite Elements in Analysis and Design, Vol. 47, pp. 1047—1057.

[8] Stancioiu, Dan; Ouyang, Huajing; Mottershead, John E. (2008), Vibration of a beam excited by a moving oscillator considering separation and reattachment, Journal of Sound and Vibration, Vol. 310, pp. 1128—1140.

[9] Stancioiu, Dan; Ouyang, Huajiang; Mottershead, John E.; James, Simon (2011) Experimental investigations of a multi-span flexible structure subjected to moving masses, Journal of Sound and Vibration, Vol. 330, pp. 2004—2016.

[10] Taheri, Michael R.; Ting, Edward C. (1990), Dynamic response of plates to moving loads: Finite element method. Computers & Structures, Vol. 34, pp. 509—521.

[11] Wagner, Nils (2000), Untersuchung der Boden-Fahrzeug-Interaktion mit gekoppelten Rand- und Finite-Element-Methoden, Dissertation Technische Universität Braunschweig.

[12] Wu, Jia-Jang (2003), Vibration of a rectangular plate undergoing forces moving along a circular path, Finite Elements in Analysis and Design, Vol. 40, pp. 41—60.

[13] Wu, Jong-Shyong; Chiang, Lieh-Kwang (2003) Out-of-plane response of a circular curved Timoshenko beam due to a moving load. International of Solids and Structures. Vol. 40, pp. 7425—7448.

[14] Yang, Y.-B.; Wu, C.-M., Yau, J.-D. (2001) Dynamic response of a horizontally curved beam subjected to vertical and horizontal moving loads. Journal of Sound and Vibration. Vol. 242, pp. 519—527.

[15] Zhao, Rui; Yu, Kaiping, (2014), Hamilton's law of variable mass system and time finite element formulations for time-varying structures based on the law, International Journal for Numerical Methods in Engineering, Vol. 99, pp. 711—736.

[16] PERMAS User's reference manual, INTES Publication No. 450

[17] VisPER User's manual, INTES Publication No. 470.

[18] http://www.intes.de